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## **Surface Vibration Pattern Induced by Underground Trains**

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### **ABSTRACT**

The prediction of vibration levels near underground trains is of growing importance for newly constructed infrastructures inside cities. A numerical FEM model has been constructed in order to predict the impact of underground trains on the surface. It is observed that the vibration amplitude pattern on the surface depends on the frequency, the shape and the material of the tunnel, and the type of terrain. The tunnel itself creates a shadow in the vertical direction at low frequencies, and radiates two lobes at high frequencies, one in the vertical direction and the other forming a certain angle with the vertical direction. The vibration amplitude as a function of the horizontal distance to the tunnel shows two bumps before decaying following the classical Barkan law. The numerical results obtained with the FEM model are compared with those obtained experimentally in a measurements campaign, and they are found to be compatible.

### **1 INTRODUCTION**

Underground trains are expanding as the preferred public transportation means in major cities. It has advantages, low visual impact and low impact on traffic, but it has also inconveniences. Particularly, trains produce vibrations that propagate through the ground to nearby buildings, affecting the human comfort. That is why it is so important to be able to predict vibration levels before the construction of such infrastructures.

One of the simplest models of propagation of vibration is the Barkan law [1]. With two parameters:  $\gamma$ , the geometrical attenuation, and  $\alpha$ , the material attenuation, one can predict the vibration amplitude at a certain distance from the source, given the vibration amplitude at a given point. The material attenuation,  $\alpha$ , depends on the type of soil, as well as on the frequency of the vibration. The geometrical attenuation,  $\gamma$ , depends on the type of source (point, line), application place (surface, buried) and the type of wave (surface waves, body waves).

A raid of measurements at a subway line in Barcelona revealed that the propagation law did not fit the Barkan law. Instead, a region of attenuated vibration appeared near the vertical direction of the tunnel.

In this study, we will focus on the vibration pattern produced on the surface by a buried line source inside a tunnel, and the attenuation that produces the tunnel itself.

In the following we first describe the 2D FEM model and give details of the simulation. Second, the model is compared with experimental measurements. Third, the results for a

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basic model with and without tunnel are discussed. Next, we study variations in depth of the basic model. Finally, some basic conclusions and future work are outlined.

## 2 MODELLING METHOD

A 2D FEM model for the tunnel-terrain set has been built that allows to simulate an infinite line source. The advantages of a 2D model over a 3D model consist in the calculus time and in the precision of the results. A 2D model allows having a smaller element size, in consequence the result can be analyzed up to higher frequencies, while at the same time it keeps the total number of elements low, well within the time and memory margins necessities for the calculation. The disadvantage of using a 2D model is that, given its symmetry, it can only simulate infinite line sources.

The concrete used for the tunnel has a density of  $\rho=2500 \text{ kg/m}^3$ , a Young modulus of  $E=27.2 \times 10^9 \text{ N/m}^2$  and a Poisson ratio of  $\nu=0.2$ , with a critical damping ratio of 0.5%.

The material used for the terrain has a density of  $\rho=1900 \text{ kg/m}^3$ , a Young modulus of  $E=90 \times 10^6 \text{ N/m}^2$  and a Poisson ratio of  $\nu=0.45$ , with a critical damping ratio of 3%.

Given these properties, one can calculate the velocities of P, S and Rayleigh waves with the following equations:

$$v_P = \sqrt{\frac{\lambda + 2G}{\rho}} = 424 \text{ m/s}; v_S = \sqrt{\frac{G}{\rho}} = 128 \text{ m/s}; v_R = v_S \cdot \frac{0.87 + 1.12\nu}{1 + \nu} = 121 \text{ m/s} \quad (1)$$

where the shear modulus,  $G$ , and the first Lamé constant,  $\lambda$ , are related to  $E$  and  $\nu$  [2].

The element size is chosen in basis of the Rayleigh waves which are the slowest ones. The element size criterion used has been to describe one Rayleigh wavelength by at least 6 elements. In order to obtain results accurate up to 80 Hz, the mesh element size is then 0.25 m. Figure 1 shows the FEM model.

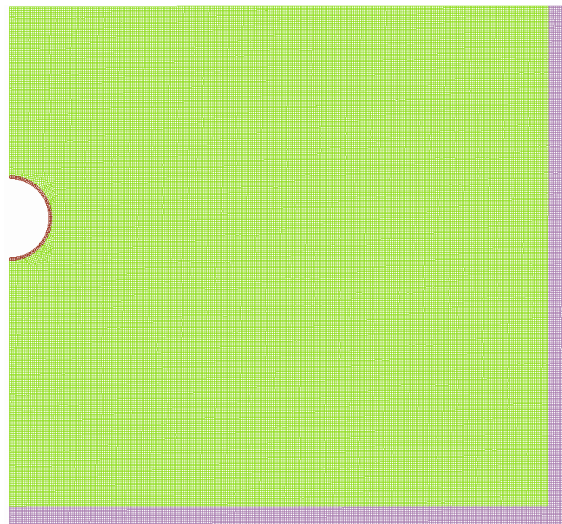


Figure 1: FEM model of the tunnel and terrain.

A vertical unitary force, constant in frequency, is applied at the center of the tunnel so that a symmetric boundary condition can be used and only half of the space has to be simulated. The terrain it has an extension at its outermost and lowest sides with a non-reflective material.

The FEM model is solved using NASTRAN and the frequency response function was calculated.

The model has been verified by comparing numerical results with known analytical conditions such as infinite line on surface and in depth with critical damping zero.

The case of infinite line surface source provides a geometrical attenuation coefficient  $\gamma=0$  for Rayleigh waves at the surface and  $\gamma=0.5$  for body waves in the interior. The case of a buried infinite line source shows also  $\gamma=0.5$  for body waves in the interior. Both simple cases are in agreement with the theoretical values [3] [4].

### 3 THE ORIGINAL TUNNEL

A raid of measurements at Barcelona suggested the possibility that the subway tunnel could be shadowing the propagation of vibration in the vertical direction.

In order to simulate this real case, the shape of the spectrum of the applied vertical force is obtained from an analytic model [5].

As a general case, it is observed that the radiation pattern on the surface strongly depends on the frequency. In consequence, the results have been analyzed here for the one-third octave bands in which trains radiate most of its energy, that is, the 40 Hz, 50 Hz and 63 Hz one-third octave bands.

Figure 2 plots the velocity amplitudes for the four measured points together with the results of the simulation for the three one-third octave bands considered. Measured velocities have been normalized to the values of the simulation. It is observed that the measured value of the velocity for the point situated at 75 m is underestimated in the simulation for the 50 Hz and 63 Hz bands. Although this can be due to the data processing since the signal to noise relation at this distance was poor.

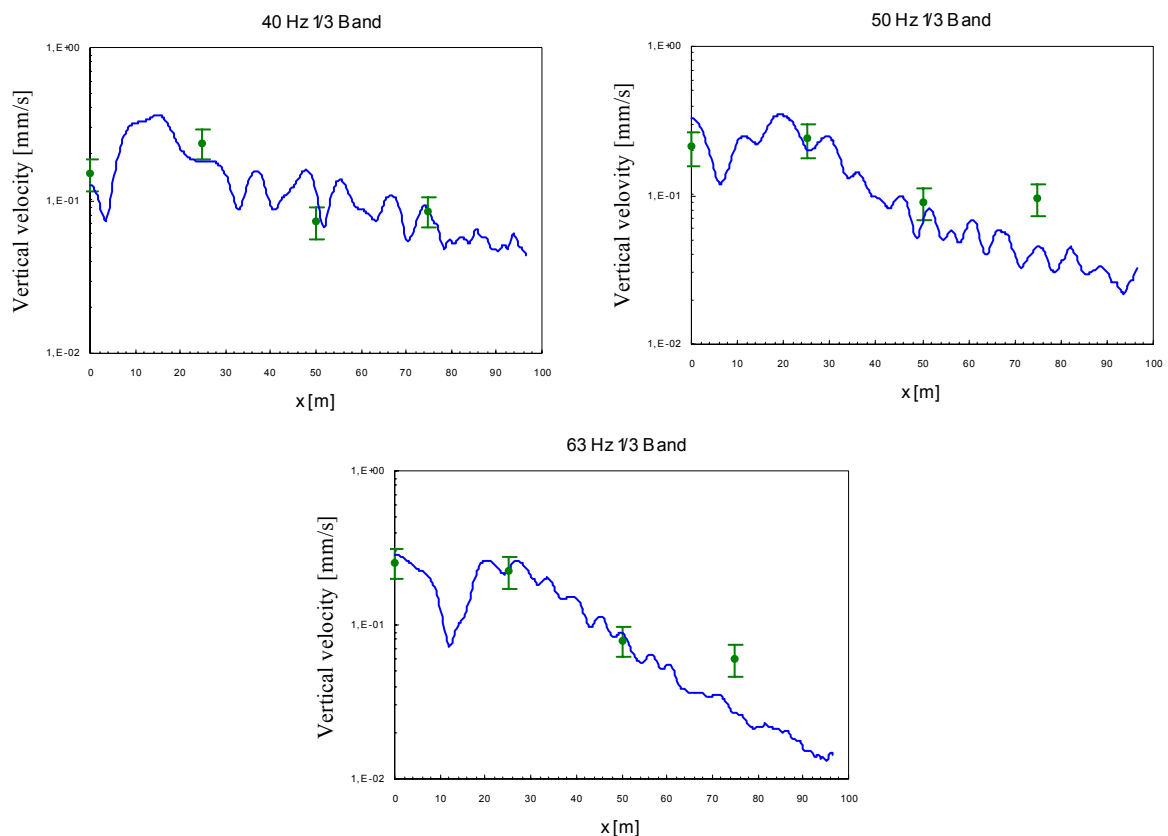


Figure 2: Comparison of the measured points (green circles) and the simulation (blue line) for the tunnel in Barcelona.

The main conclusion from the simulation is that it agrees with the measurements in the fact that vibration amplitudes in the vertical of the tunnel can be lower than at a certain angle, in spite of being closer to the source. This behavior is not possible with the Barkan law for positive values of  $\alpha$  and  $\gamma$ . Negative values have no physical sense since the Barkan law accounts only for geometrical and material attenuation, never amplification.

#### 4 CASES OF STUDY

For the sake of simplicity, a simpler model with a vertical force constant in frequency is discussed in the following. Variations of this simple model at different depths are also studied.

##### 4.1 The basic model

The basic model is composed of a tunnel of 11 m in diameter with the base situated at a depth of 35 m, and a terrain that extends 95 m in the horizontal direction and 90 m in the vertical direction.

This model is compared with an infinite buried line without tunnel. It is interesting to study a buried infinite line without a tunnel to see the attenuation caused by the tunnel itself. Figure 3 represents the vibration amplitude of a buried infinite line without tunnel, fitted by a Barkan law, together with the corresponding buried infinite line inside a tunnel. In this figure, the abscissa axis is the distance to the source, which is the parameter that appears in the Barkan equation.

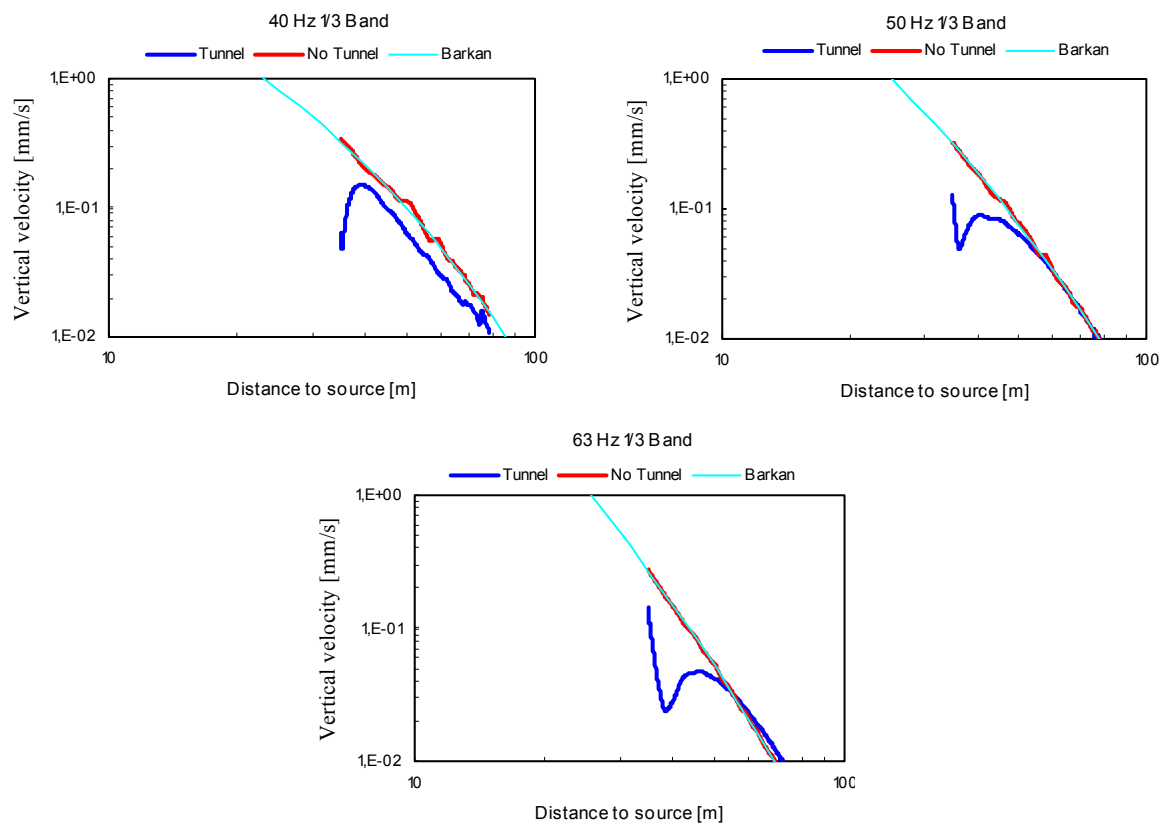


Figure 3: Surface vibration amplitude as a function of the distance to the source for a buried infinite line inside a tunnel (dark blue), a buried infinite line without a tunnel (red) and its Barkan law fit (light blue).

As it can be seen, the buried line without tunnel follows precisely a Barkan law. On the contrary, the buried line inside a tunnel does not follow a Barkan law at distances close to the vertical of the tunnel. It only follows it outside the shadowed region, that is, far from the vertical of the tunnel.

The shadowed region can be appreciated better in figure 4 which shows the attenuation, in dB, caused by the tunnel as a function of the distance to the vertical of the tunnel.

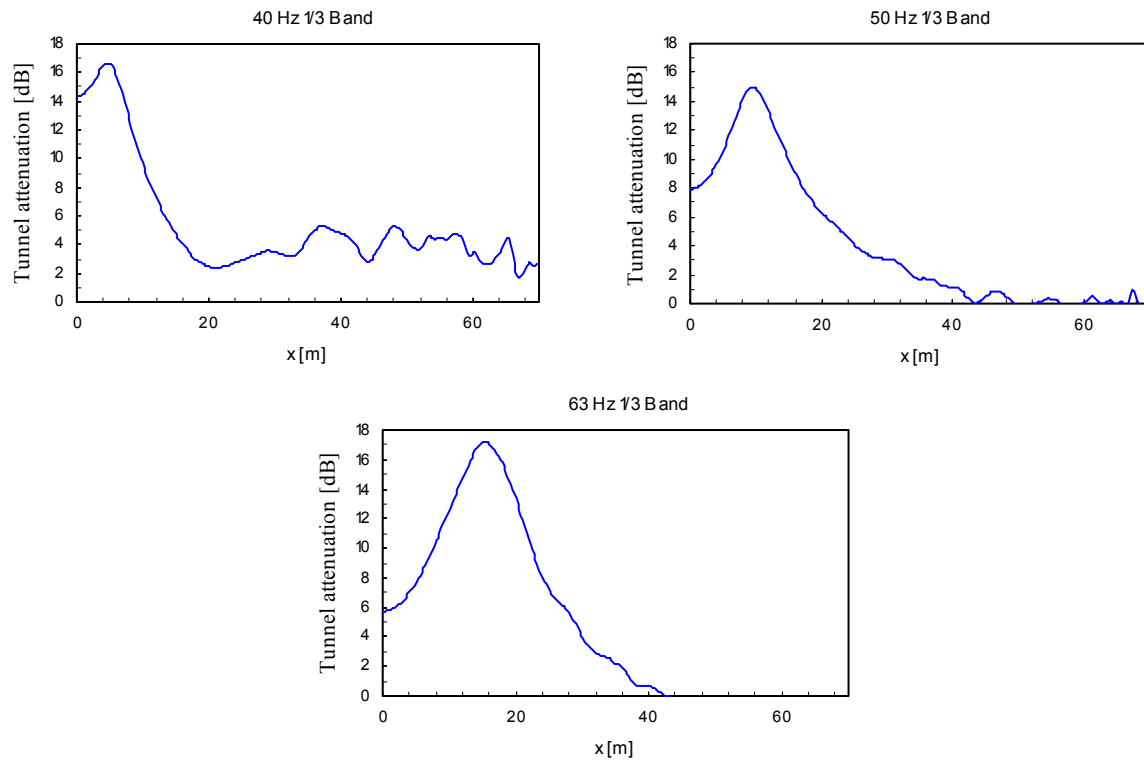


Figure 4: Attenuation in dB caused by the tunnel as a function of the distance to the vertical of the tunnel.

It can be observed that the vibration is attenuated in the vertical direction of the tunnel. But the maximum attenuation is achieved at a certain angle from the vertical of the tunnel. That angle is bigger for higher frequencies. Attenuation in the shadowed region can be as high as 17 dB.

Figure 5 shows with more detail the radiation pattern emerging from the tunnel. Again, one can see that at low frequencies the tunnel casts a shadow in the vertical direction so that the vibration amplitude on the surface is higher at a certain angle from the vertical than in the vertical direction, in spite of being closer to the source. As the frequency grows the radiation lobe in the vertical direction gets more and more important, while the secondary lobe loses amplitude.

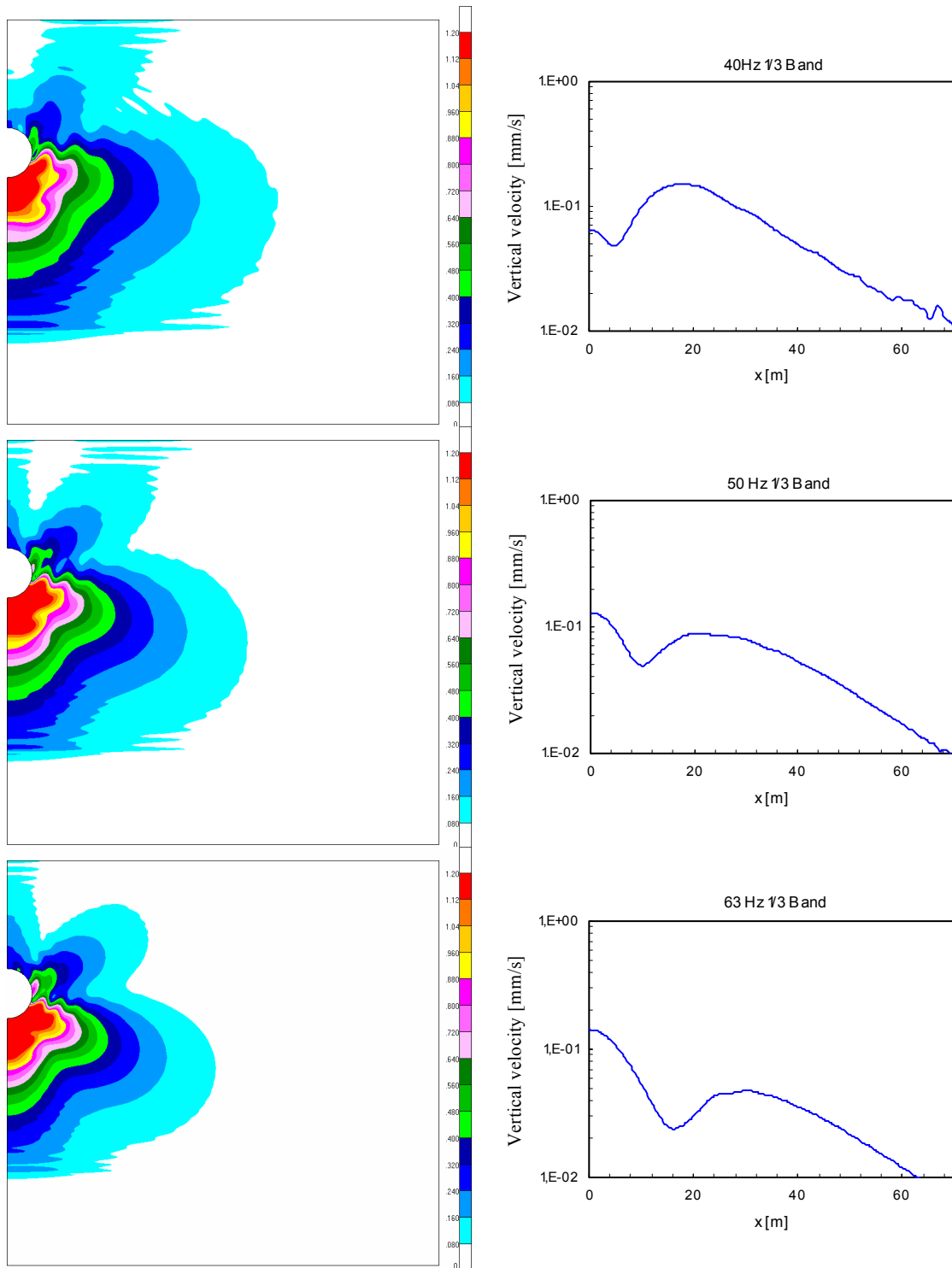


Figure 5: Figures on the left column show velocity amplitude at each point of the model for the one-third octave bands centered on 40 Hz, 50 Hz and 63 Hz. Figures on the right show vibration amplitude at surface level as a function of the distance,  $x$ , to the vertical of the tunnel for the same one-third octave bands.

It has to be said that the radiation pattern, with four emission lobes observed around the tunnel, is not a direct consequence of the vibration normal modes of the tunnel-terrain set, since at these frequencies there is a very high density of modes, nor to the vibration modes of the tunnel itself. Figure 6 shows the fourth and fifth normal modes of the tunnel situated at 43.3 Hz and 63 Hz respectively. It can be appreciated that these vibration modes present six and seven maximums, instead of the four lobes seen in figure 5.

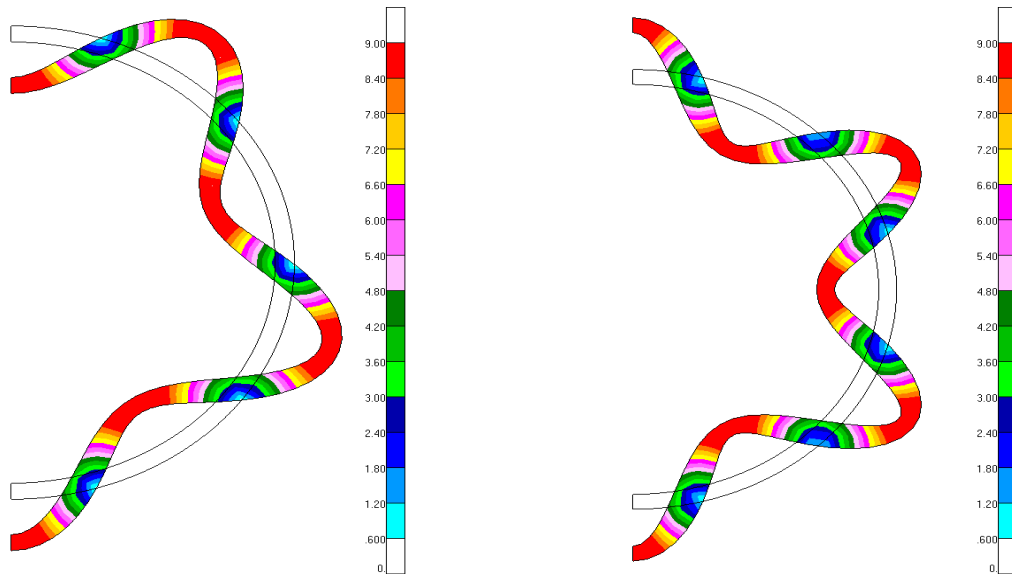


Figure 6: Normal modes of the tunnel. Left: 4<sup>th</sup> normal mode at 43.3 Hz. Right: 5<sup>th</sup> normal mode at 63 Hz.

#### 4.2 Different depths

Underground trains circulate at a wide range of depths. In this subsection we study the effect of emplacing the tunnel at different depths. Figure 7 shows the vibration pattern on surface in the 40 Hz one-third octave band for tunnels of 11 m in diameter at depths between 25 m and 60 m. It can be appreciated how the overall shape of the vibration amplitude as a function of the distance to the vertical of the tunnel is the same, but the position of the maximum is displaced to higher distances as the source goes deeper.

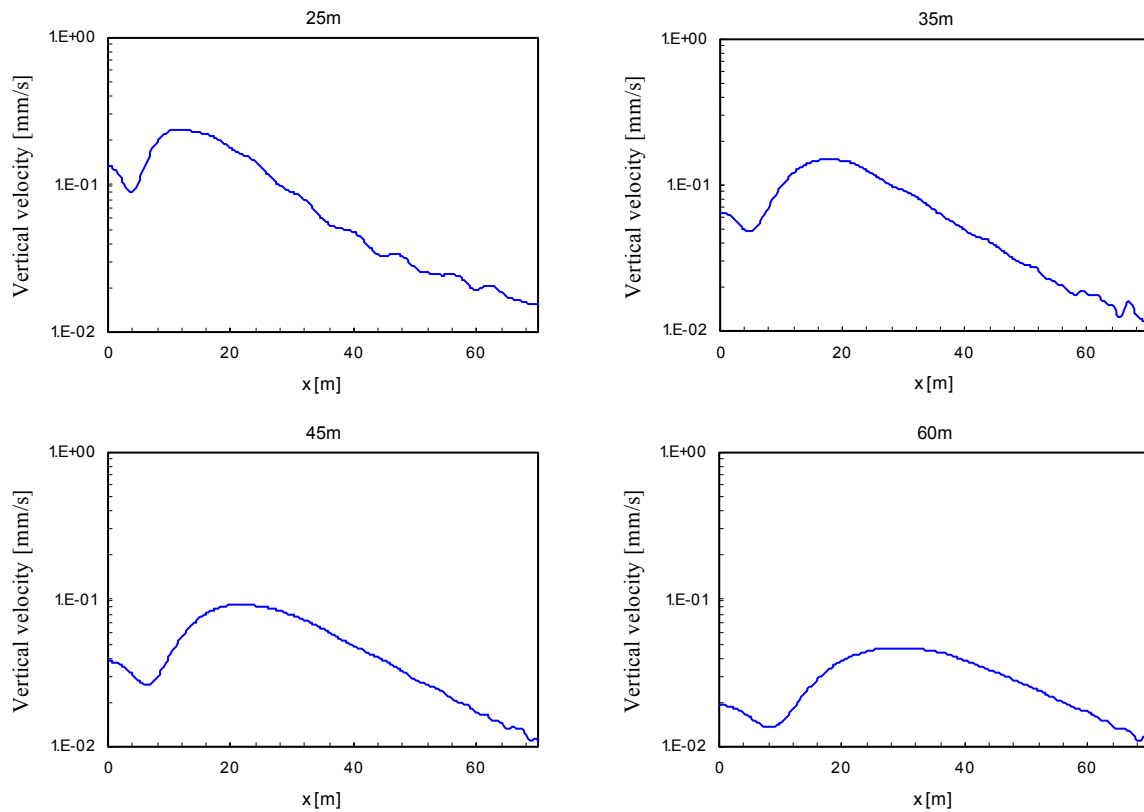


Figure 7: Vibration pattern on surface for an 11 m in diameter tunnel situated at 25 m, 35 m, 45 m and 60 m of depth in the 40 Hz one-third octave band.

Table 1 shows that the angle measured from the vertical to the position of this second vibration lobe,  $\theta$  from here on, is almost constant for each one-third octave as a function of the source depth.

Table 1: Emission angle of the second lobe ( $\theta$ ) for each tunnel depth and one-third octave band.

Depth	One-third octave bands		
	40 Hz	50 Hz	63 Hz
25 m	24.2°	31.8°	39.8°
35 m	27.2°	30.2°	40.6°
45 m	25.2°	29.6°	40.5°
60 m	26.2°	28.8°	37.6°

For each one-third octave band, the mean  $\theta$  angles are:  $\theta_{40} = 25.7^\circ \pm 1.3^\circ$ ;  $\theta_{50} = 30.1^\circ \pm 1.3^\circ$ ;  $\theta_{63} = 39.6^\circ \pm 1.4^\circ$ . Being the mesh size of 250 mm, it makes the resolution for  $\theta$  of about  $0.5^\circ$  for the range of  $x$  and depths studied here. So,  $\theta$  can really be considered as a constant.

The angle of the minimum, that is, the maximum attenuation angle, is also constant in depth. However, the knowledge of  $\theta$  is more interesting than the angle of maximum attenuation because from this angle on, the propagation follows the classical Barkan law, as seen in figure 3. The vibration levels at distances higher than  $depth \cdot \tan \theta$  from the vertical of the tunnel can be predicted with the material damping part of the Barkan equation:

$$v_2 = v_1 \left( \frac{r_1}{r_2} \right)^\gamma e^{-\alpha(r_2 - r_1)} \quad (2)$$

Far away from the source the term of material damping,  $\alpha$ , dominates and the geometrical attenuation,  $\gamma$  term, is of second order. That is, good fits can be obtained for a wide range of  $\gamma$ . In other words, the determination of  $\gamma$  fitting the vibration amplitude curve has a large uncertainty.

Being  $\theta$  independent from the depth means that the propagation of vibration in the terrain is straightforward. This is because it has been used a homogeneous non-stratified terrain. The surface vibration pattern is then due to a local phenomenon at the position of the tunnel, that is, a diffractive phenomenon.

## 5 CONCLUSIONS

A simple numerical 2D FEM model for a tunnel-terrain set has been presented here. The aim was to analyze the surface vibration pattern provoked by an underground train. This model has been tested with known theoretical solutions as well as with field measures.

It has been shown that the tunnel blocks the outgoing radiation at a certain angle proportional to the frequency of the vibration. The propagation law outside the shadowed region follows the classical Barkan equation.

The overall shape of the surface vibration pattern does not change with the depth of the tunnel; it is only seen more widespread, implying that the shadow is produced by a local phenomenon of interaction of the vibration wave with the tunnel, like diffraction. It has been possible to isolate the phenomenon thanks to the use of a very simple model with a homogeneous non-stratified terrain and a force with a constant spectrum.

We are now in the process of characterizing the shadow for tunnels of different shapes and materials, which undoubtedly affect the surface vibration pattern.



## 6 ACKNOWLEDGEMENTS

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