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TWO STEP OPTIMIZATION OF SECONDARY SOURCES AND ERROR TRANSDUCERS IN GLOBAL ACTIVE NOISE CONTROL IN ENCLOSURES

J. Ignacio Palacios

SENER Ingeniería y Sistemas, 392 Provença St., 08025, Barcelona, Spain
email: jignacio.palacios@sener.es

Jordi Romeu, Andreu Balastegui

Acoustic and Mechanical Engineering Laboratory, Mechanical Engineering Department, Universitat Politècnica de Catalunya, Colom 11, 08222 Terrassa, Spain
e-mail: jordi.romeu@upc.edu

Global active control of sound can be achieved inside enclosures under low modal acoustic fields. However, the performance of the system depends largely on the localization of the elements of the control system. For a purely acoustic active control system, in which secondary acoustic sources (loudspeakers) and pressure transducers as error sensors are used, several optimisation strategies have been proposed. These strategies usually rely on partial approximation to the problem, focussing on the study of number and localisation of secondary sources, without considering error transducers, or selecting the best positions of secondary sources and error transducers of an initial set of candidate locations for these elements. The strategy presented here is based on two steps: the first is rather common with this sort of problem, and its goal is to find the best locations for secondary sources and their strengths by minimising the potential energy of the enclosure. The second step is the localisation of the error transducer which assures the results of the first step. It has been analytically demonstrated that the optimum location of error transducers is at the minimum of the optimally attenuated acoustic field. Numerical validation of this principle is carried out in a parallelipedic enclosure.

1. Introduction

Global active noise control in enclosures can be defined as that system which allows us to obtain a reduction in sound pressure throughout the enclosure. An approximation to this global attenuation can be found by a reduction of the acoustic potential energy, defined as

$$E_p = \frac{1}{4\rho_0 c_0^2}\int_V |\, p(x)\,|^2 \, dV \tag{1}$$

However, practical applications of such kind of control system would require a great number of error sensors. Thus, the acoustic potential energy is estimated by the following cost function:

$$J_p = \frac{V}{4\rho_0 c_0^2 L}\sum_{l=1}^{L} | p(x_l) |^2 \tag{2}$$

Where L means the quantity of error sensors and $p(x_l)$ is the error measured signal. In active control of sound, a set of secondary sources (loudspeakers) generate a secondary sound field which must interfere with the primary sound field in order to attenuate the acoustic potential energy, but previous works have demonstrated that results depend largely on the localization of the secondary source and its relationship with the sound field. On the other hand, most active control systems are based on adaptive algorithms which minimize a set of error signals, and it is clear that these error signals must be representative of the acoustic potential energy. As a result, the design of an active noise control to obtain global attenuation inside an enclosure must optimize the location of the sources and the error transducers.

In global active control of sound in enclosures, and for a single set of secondary sources, there is only one set of secondary strengths which minimizes the potential energy[1]. However, the amplitude of the residual sound field (and its potential energy) depends on the location of the secondary sources. If they are not placed in optimal positions, poor attenuation[2] or increments of the sound field will be obtained[3]. Some rules can be followed in order to optimize the location of the secondary sources: in order to cancel dominant modes with minimum effort, secondary sources should be placed at points where these modes have maximum values[4,5,6]. One simplified approximation to that rule is to locate secondary sources in the corners of the enclosure in order to make them able to excite most of the acoustic modes[7,8,9]. With this set up, results could be non optimal, but it is considered a good compromise between results and the number or secondary sources, even in low modal density[10,11,12]. However, especially in case of off-resonance, results can be far from optimal. Thus, optimization methods of location and strengths of secondary sources have been proposed in order to minimize the potential energy, usually estimated by the cost function[4] applied to a mesh of measured or calculated points.

The optimal position and flow of the sources can be found through different algorithms, generally reaching conclusions similar to those established by[4]. However, for the realistic prediction of the final acoustic field it is necessary to include the effect of the error microphones. In this case, the magnitude to minimize is J_p. Several approaches are possible. The most common one is to start from a set of possible fixed positions of secondary sources and, using an optimization model, to determine the number and position of the secondary sources which minimize J_p (without this necessarily being representative of E_p). The transfer functions can be analytically determined for regular enclosures[13,14,2], through finite[10,15] or boundary elements[16,17] methods, experimentally[18,19,20] or by means of a combination of several techniques[21]. However, when using experimental data the results show a strong dependency to the conditions of the test, which would condition the results. Generally, optimization allows for a reduction in the number of sources with respect to the initial selection, but also it reaches the conclusion that the areas in which the secondary sources are most efficient correspond to the zones with maximums from the primary field[10], which is consistent with the selection criteria[4]. Nevertheless, an active control system made up of a relatively high number of secondary sources and error microphones might be required, even in low modal density[10,11,12].

On many occasions the reduction in J_p does not guarantee a reduction in E_p, given that the criteria of minimisation of J_p identifies more with a minimisation of pressures than of E_p[4,17,10,22,19]. In these cases, it has been proven that the attenuation of J_p does not necessarily entail attenuation of E_p in spite of being within the range of low modal density[23,17,24,25,7]. In the case of strategies ASAC, primary field attenuation can also be realised with the aim of reducing J_p which depends on structural vibration. In this case, either accelerometers[26,27,28,29,2] can be used or distributed PVDF sensors[30,31,32], with the aim of normally reducing the radiation capacity of the vibrating surfaces, which could be translated as a reduction of E_p in the enclosure. Generally the results obtained using a J_p based on structural sensors do not improve on those obtained using microphones as error sensors[33,22,28,34].

The optimization in the position of error sensors has not received much interest, although strategies for the location of error sensors through optimization algorithms have also been proposed[24]. Some criteria for location deduced from numerical models are found, like, for example, that error sensors must be located in a way that they avoid the nodes of dominant modes[4] and that, preferably, they are located at points of maximum sound pressure to detect the predominant modes[17,4]. The minimisation of J_p calculated from microphones located in the corners of the enclosure[4] can give rise to near optimal results. Other works have found that it is advisable to locate the error sensors in positions of maximum difference between the primary field and the field of optimal attenuation, at least in cases in which the primary field consists of a predominant mode[2]. A similar criterion, also obtained from the observation of attenuated field modelling is that the best location for the error microphone is at the optimally attenuated field's minimum[7].

More recent strategies define the cost function using the energy density, which is a local variable that adds potential energy and kinetic energy[35]. This strategy is less sensitive to the position of the error transducer[36] and gives generally better results in global reduction from the cancellation of the energy density to a point in respect to the cancellation of sound pressure at that same point[7]. Fewer error sensors[37,38] are consequently required, although it must be taken into account that the energy density sensors themselves are made up of six microphones[37].

2. The minimisation of acoustic potential energy with 1 error microphone in its optimal position

Two step optimization procedure means that optimal location of the secondary source will be firstly demonstrated. After that, optimal position of error microphone which ensures the optimal attenuation will be deduced.

Sound pressure at any point within the enclosure due to one primary and one secondary source can be written as:

$$p(x_0) = \psi_0(x)\cdot\left[\left(\psi(y_1)\,q_1 + \psi(y_2)\,q_2\right)\right] = \psi_0(x)\cdot \boldsymbol{V} \tag{1}$$

Where $\boldsymbol{\psi_0}$(x), $\boldsymbol{\psi}$(y_1) y $\boldsymbol{\psi}$(y_2) are complex vectors and **V** is the one which could be called generating vector, also complex. The potential energy, on the other hand, can be written in a synthetic way according to:

$$Ep = \left[\left(\psi(y_1)\,q_1 + \psi(y_2)\,q_2\right)\right]^2 \tag{2}$$

In order to find q_2 which reduces E_p, we must derive E_p with respect to q_2 and equal to zero:

$$\frac{dEp}{dq_2} = 2\,\psi(y_2)\left[\left(\psi(y_1)\,q_1 + \psi(y_2)\,q_2\right)\right] = 0 \tag{3}$$

One solution consists in the vectorial sum of the interior of the parenthesis being null. This implies:

$$\psi(y_1)\,q_1 = -\,\psi(y_2)\,q_2 \tag{4}$$

This result means that $\boldsymbol{\psi}(y_1)$ e $\boldsymbol{\psi}(y_2)$ are parallel. The most obvious case is that the position of the primary and secondary source coincides. It is possible on the other hand to annul the complete dot product, which gives the following result:

$$q_{2opt} = -\frac{\boldsymbol{\psi}(y_2)\cdot\boldsymbol{\psi}(y_1)}{\boldsymbol{\psi}(y_2)\cdot\boldsymbol{\psi}(y_2)} q_1 \tag{5}$$

In this case the optimal strenght for any position of the secondary source turns out to be the projection of the generating vector $\boldsymbol{\psi}(y_1)q_1$ in the direction of the vector $\boldsymbol{\psi}(y_2)$. This projection is, on the other hand, the best approximation of $\boldsymbol{\psi}(y_2)$ and $\boldsymbol{\psi}(y_1)$ according to Bessel's property of inequality. Now, according to Cauchy-Schwarz's inequality, the result of q_{2opt} can be written as a product of modules, resulting in:

$$q_{2opt} = -\frac{\psi(y_1)\cos\theta}{\psi(y_2)} q_1 \tag{6}$$

Then the secondary field generating vector $\boldsymbol{\psi}(y_2)q_{2opt}$ is equal to, as has been indicated, the projection of the generating vector of the primary field in the direction of the secondary field generating vector.

$$\boldsymbol{\psi}(y_2)q_{2opt} = \psi(y_1)q_1\cos\theta\ \boldsymbol{u_2} \tag{7}$$

Where $\mathbf{u_2}$ is the unitary vector in the direction of $\boldsymbol{\psi}(y_2)$. This means that the more coupled the vectors $\boldsymbol{\psi}(y_1)$ y $\boldsymbol{\psi}(y_2)$ the more cancellation we will have and in the ideal case that $\boldsymbol{\psi}(y_1) = \boldsymbol{\psi}(y_2)$, the E_p will be zero, because the secondary field would replicate exactly the primary field. Then, given a primary source in an enclosure, its $\boldsymbol{\psi}(y_1)$ can be numerically calculated and then, a systematic calculation for the $\cos\theta$ according to:

$$\cos\theta = \frac{\boldsymbol{\psi}(y_1)\cdot\boldsymbol{\psi}(y_2)}{\psi(y_1)\psi(y_2)} \tag{8}$$

can be carried out for all or some of the positions where the secondary source can be located. The maximum value would eventually give the optimum location of the secondary source.

After the secondary source is optimally located, we again write the expression of sound pressure at any point, after applying active control, with q_{2opt} applied to the secondary source:

$$p(x_0) = \boldsymbol{\varphi}(x)\cdot\left[\left(\boldsymbol{\psi}(y_1)\, q_1 + \boldsymbol{\psi}(y_2)\, q_{2opt}\right)\right] = \boldsymbol{\varphi}(x)\cdot\boldsymbol{V_{opt}} \tag{9}$$

According to the orthogonal condition of vectors, vector $\boldsymbol{\psi}(y_2)q_{2opt}$ can decompose in the direction of $\mathbf{u_1}$ and in its perpendicular according to the following relation:

$$\boldsymbol{\psi}(y_2)\, q_{2opt} = \left(\psi(y_2)\, q_{2opt}\cos\theta\right)\boldsymbol{u_1} + \left(\psi(y_2)\, q_{2opt} - \psi(y_2)\, q_{2opt}\cos\theta\right)\boldsymbol{u_{\perp 1}} \tag{10}$$

If the value q_{2opt} is substituted and then, the $\mathbf{V_{opt}}$ vector is written based on the components of the vector in the direction of u_1 and in its perpendicular:

$$\boldsymbol{V_{opt}} = \psi(y_1)\, q_1\left[\left(1-\cos^2\theta\right)\boldsymbol{u_1} + \left(\cos^2\theta - \cos\theta\right)\boldsymbol{u_{\perp 1}}\right] \tag{11}$$

The pressure at the point of the attenuated field is therefore:

$$p(x_0) = \boldsymbol{u_0}(x)\cdot\left[\left(1-\cos^2\theta\right)\boldsymbol{u_1} + \left(\cos^2\theta - \cos\theta\right)\boldsymbol{u_{\perp 1}}\right]\psi_0(x)\psi(y_1)\,q_1 \tag{12}$$

In this field, a point of null pressure will exist if $\mathbf{u_0}$(x) is orthogonal to **V**, meaning that the components of $\mathbf{u_0}$(x) must be:

$$\boldsymbol{u_0}(x) = \left[\left(\cos^2\theta - \cos\theta\right)\boldsymbol{u_1} - \left(1-\cos^2\theta\right)\boldsymbol{u_{\perp 1}}\right] \tag{13}$$

In these circumstances, it is possible to state that if a point of null pressure exists in the minimized field, the cancellation of the pressure at that point guarantees that the applied strength q is the optimal one, because it will give a generating vector **V** orthogonal at $\boldsymbol{\psi_0}$(x).

3. Calculations

For this study a rectangular cavity will be used with the characteristics outlined in table 1.

Table 1: Physical characteristics

Cavity properties	
Dimensions	L_x=1,450 m
	L_y=1,150 m
	L_z=1,050 m
Density	ρ_0=1,22 kg/m^3
Damping	ξ=0,005

Wall properties	
Thickness	t=0,02 m
Young modulus	$E=2\cdot10^9$ Pa
Poisson modulus	ν=0,2
Density	ρ=690 kg/m^3
Damping	ξ=0,02

Source properties	
Primary source location	*(0.1, 0.1, 0.1)*
Primary source strenght	*1 m3/s*
Secondary source location	*(1.35,1.05,0.1)*

As it has benn shown, it is possible to locate a position for the error microphone which provides a final acoustic field equivalent to the final acoustic field in the case of ideal active control. Figure 1 shows the inverse value of acoustic potential energy (1/E_p) optimally obtained by the different positions of the error microphone at 150 Hertz on the XY plane with z=0,40 m. The optimal position for the error microphone which minimizes the value of the cost function of J_p at 150 Hertz is obtained in this way and it is found to be at (0.65, 1.15, 0.4) m from the origin. Figure 2 shows the primary acoustic field and final acoustic field after the optimum active control in the cavity walls at a frequency of 150 Hertz.

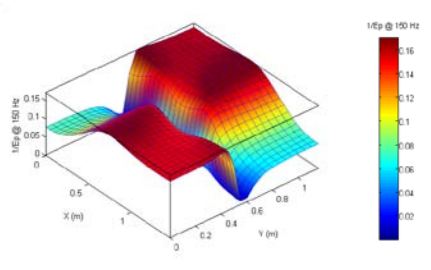

Figure 1: Graph showing the obtainable inverse value of the potential energy (1/E_p) at 150 Hertz for different error microphone locations in the cut at z=0,40 m (left) (J^{-1})

It is, on the other hand, possible to realise a comparison between the minimum acoustic potential energy of the actively controllated sound field level for each location of the error microphone and the amplitude of the ideal acoustic field after applying an optimal secondary strength. It is possible to observe that a relation exists between the lowest sound pressure (Fig. 3, left) at the optimally attenuated sound field and the optimal positions for the error microphone (positions with a minimal level of potential acoustic energy, Fig. 3, right).

In this way, as predicted in previous section, it can be observed that the locations of the error microphone which minimize the potential energy in the cavity (minimum in the representation of the acoustic potential energy) agree with the positions with minimum levels of sound pressure for an optimum actively attenuated sound field.

The comparison of the optimally attenuated sound field (Fig 3, left) and the attenuated sound field considering the best location for the error microphone (Fig, 2, right) shows almost the same pattern (the scale is not the same). The average attenuation in the cavity at 150 Hertz is 18 dB in both cases. Figure 4 shows the modal participation factors obtained for the ideal case of minimisation of acoustic potential energy (without considering the effect of error microphone and using an optimal secondary strength) and the modal participation factors obtained in this case at 150 Hertz. This shows a great similarity in the results obtained in both cases.

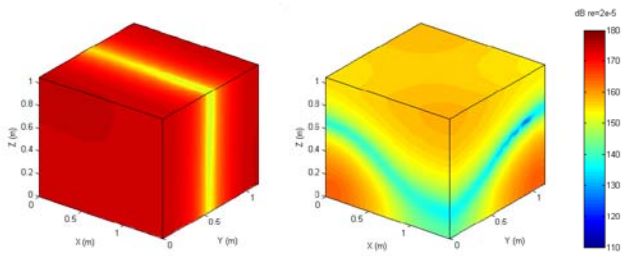

Figure 2: primary acoustic field (left) and final (right) after optimum location of error microphone at 150 Hertz (dB re=2e-5)

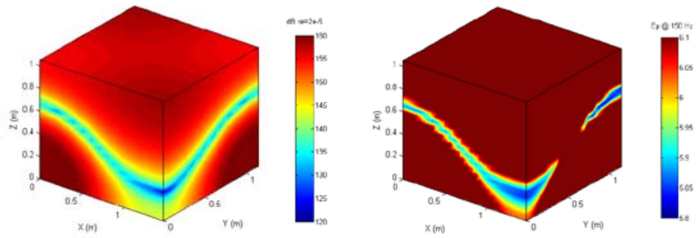

Figure 3: Acoustic field after the minimization of the acoustic field at 150 Hertz (dB re=2e-5, left) and the value of potential acoustic energy associated to each position of the error microphone at 150 Hertz (J, right)

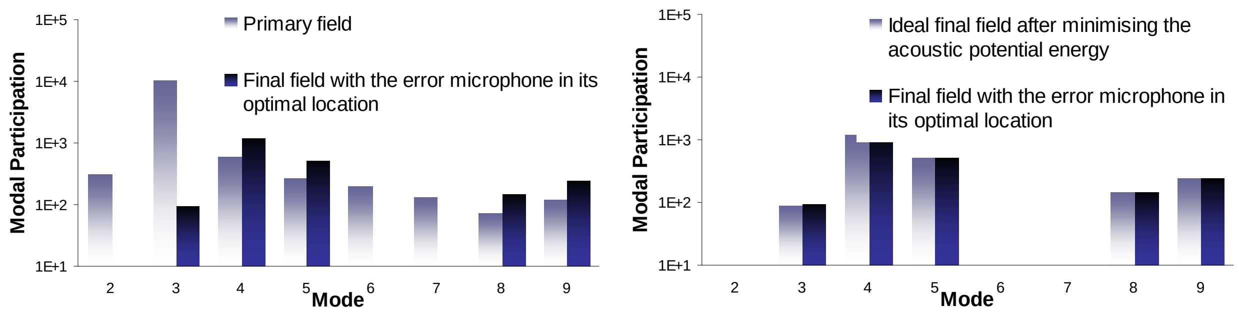

Figure 4: Factors of modal participation after an ideal minimisation of the acoustic potential energy and with 1 error microphone in a real case at 150 Hertz

4. Conclusions

A two step optimization of single input single output active sound control is presented and demonstrated from the theoretical point of view. Firstly, the optimum strength of the secondary source must be obtained by known methods, and secondly, the error microphone must be placed in the minimum of the optimally attenuated sound field in order to ensure the optimum attenuation. This statement have been theoretically demonstrated and confirmed by numerical calculations carried out in a parallelepipedic cavity.

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